UNIT -5
Similarity Relations and Performance Characteristics

Topics

1. Centrifugal Pumps- Main Components, Working
2. Priming
3. Work done and Velocity Triangle
4. Heads in Centrifugal Pumps
5. Efficiency
6. Characteristics Curves
7. Multistage Pumps
8. Characteristics Curves
9. Axial Flow Pumps

Topics

- **Unit quantities**
- **Specific speed**
- **Cavitation**
- **Thoma's cavitation number**
- **Net positive suction head**

Unit quantities

In order to predict the behaviour of a turbine working under varying conditions of head, speed, output and gate opening, the results are expressed in terms of quantities which may be obtained when the head on the turbine is reduced to unity. The conditions of the turbine under unit head are such that the efficiency of the turbine remains unaffected. The following are the three important unit quantities which must be studied under unit head :

1. Unit speed, 2. Unit discharge, and 3. Unit power.

18.12.1 Unit Speed. It is defined as the speed of a turbine working under a unit head (*i.e.*, under a head of 1 m). It is denoted by 'N_u'. The expression for unit speed (N_u) is obtained as :

Let N = Speed of a turbine under a head H,

H = Head under which a turbine is working,

u = Tangential velocity.

The tangential velocity, absolute velocity of water and head on the turbine are related as

$$u \propto V, \quad \text{where } V \propto \sqrt{H}$$

$$\propto \sqrt{H} \quad \ldots(i)$$

Also tangential velocity (u) is given by

$$u = \frac{\pi D N}{60}, \quad \text{where } D = \text{Diameter of turbine.}$$

For a given turbine, the diameter (D) is constant.

$$\therefore \quad u \propto N \text{ or } N \propto u \text{ or } N \propto \sqrt{H} \quad \left(\because \text{ From } (i),\ u \propto \sqrt{H}\right)$$

$$\therefore \quad N = K_1 \sqrt{H} \quad \ldots(ii)$$

where K_1 is a constant of proportionality.

If head on the turbine becomes unity, the speed becomes unit speed or

when $H = 1, N = N_u$

Substituting these values in equation (*ii*), we get

$$N_u = K_1 \sqrt{1.0} = K_1$$

Substituting the value of K_1 in equation (*ii*),

$$N = N_u \sqrt{H} \text{ or } N_u = \frac{N}{\sqrt{H}}. \quad \ldots(18.29)$$

Unit Discharge

The discharge passing through a given turbine under a head 'H' is given by,

$$Q = \text{Area of flow} \times \text{Velocity}$$

But for a turbine, area of flow is constant and velocity is proportional to $\sqrt{H}$.

$$\therefore \quad Q \propto \text{Velocity} \propto \sqrt{H}$$

or

$$Q = K_2 \sqrt{H} \qquad \ldots(iii)$$

where K_2 is constant of proportionality.

If $H = 1$, $Q = Q_u$. (By definition)

Substituting these values in equation (*iii*), we get

$$Q_u = K_2 \sqrt{1.0} = K_2.$$

Substituting the value of K_2 in equation (*iii*), we get

$$Q = Q_u \sqrt{H}$$

$$\therefore \quad Q_u = \frac{Q}{\sqrt{H}} \qquad \ldots(18.30)$$

Unit quantities

18.12.3 Unit Power. It is defined as the power developed by a turbine, working under a unit head (*i.e.*, under a head or 1 m). It is denoted by the symbol 'P_u'. The expression for unit power is obtained as :

Let H = Head of water on the turbine,

P = Power developed by the turbine under a head of H,

Q = Discharge through turbine under a head H.

The overall efficiency (η_o) is given as

$$\eta_o = \frac{\text{Power developed}}{\text{Water power}} = \frac{P}{\frac{\rho \times g \times Q \times H}{1000}}$$

$$\therefore \quad P = \eta_o \times \frac{\rho \times g \times Q \times H}{1000}$$

$$\propto Q \times H$$

$$\propto \sqrt{H} \times H \qquad (\because \quad Q \propto \sqrt{H})$$

$$\propto H^{3/2}$$

$$\therefore \quad P = K_3 H^{3/2} \qquad \text{...}(iv)$$

where K_3 is a constant of proportionality.

When $H = 1$ m $P = P_u$

$$\therefore \quad P_u = K_3 (1)^{3/2} = K_3.$$

Substituting the value of K_3 in equation (iv), we get

$$P = P_u H^{3/2}$$

$$\therefore \quad P_u = \frac{P}{H^{3/2}}. \qquad \text{...(18.31)}$$

Specific speed

▶ 18.11 SPECIFIC SPEED

It is defined as the speed of a turbine which is identical in shape, geometrical dimensions, blade angles, gate opening etc., with the actual turbine but of such a size that it will develop unit power when working under unit head. It is denoted by the symbol N_s. The specific speed is used in comparing the different types of turbines as every type of turbine has different specific speed.

In M.K.S. units, unit power is taken as one horse power and unit head as one metre. But in S.I. units, unit power is taken as one kilowatt and unit head as one metre.

18.11.1 Derivation of the Specific Speed. The overall efficiency (η_o) of any turbine is given by,

$$\eta_o = \frac{\text{Shaft power}}{\text{Water power}} = \frac{\text{Power developed}}{\frac{\rho \times g \times Q \times H}{1000}} = \frac{P}{\frac{\rho \times g \times Q \times H}{1000}} \quad \ldots(i)$$

where H = Head under which the turbine is working,

Q = Discharge through turbine,

P = Power developed or shaft power.

From equation (i),

$$P = \eta_o \times \frac{\rho \times g \times Q \times H}{1000}$$

$$\propto Q \times H \text{ (as } \eta_o \text{ and } \rho \text{ are constant)} \quad \ldots(ii)$$

Now let

D = Diameter of actual turbine,

N = Speed of actual turbine,

u = Tangential velocity of the turbine,

N_s = Specific speed of the turbine,

V = Absolute velocity of water.

The absolute velocity, tangential velocity and head on the turbine are related as,

$$u \propto V, \text{ where } V \propto \sqrt{H}$$

Specific speed

$$u \propto V, \text{ where } V \propto \sqrt{H}$$

$$\propto \sqrt{H} \quad \text{...(iii)}$$

But the tangential velocity u is given by

$$u = \frac{\pi DN}{60}$$

$$\propto DN \quad \text{...(iv)}$$

$\therefore$ From equations (*iii*) and (*iv*), we have

$$\sqrt{H} \propto DN \text{ or } D \propto \frac{\sqrt{H}}{N} \quad \text{...(v)}$$

The discharge through turbine is given by

$$Q = \text{Area} \times \text{Velocity}$$

But $\text{Area} \propto B \times D$ (where B = Width)

$$\propto D^2 \quad (\because \; B \propto D)$$

And $\text{Velocity} \propto \sqrt{H}$

$\therefore$ $Q \propto D^2 \times \sqrt{H}$

Specific speed

$$\propto \left(\frac{\sqrt{H}}{N}\right)^2 \times \sqrt{H} \qquad \left(\because \text{ From equation } (v),\ D \propto \frac{\sqrt{H}}{N}\right)$$

$$\propto \frac{H}{N^2} \times \sqrt{H} \propto \frac{H^{3/2}}{N^2} \qquad \ldots(vi)$$

Substituting the value of Q in equation (ii), we get

$$P \propto \frac{H^{3/2}}{N^2} \times H \propto \frac{H^{5/2}}{N^2}$$

$$\therefore \qquad P = K\frac{H^{5/2}}{N^2}, \text{ where } K = \text{Constant of proportionality.}$$

If $P = 1$, $H = 1$, the speed N = Specific speed N_s. Substituting these values in the above equation, we get

$$1 = \frac{K \times 1^{5/2}}{N_s^2} \qquad \text{or} \qquad N_s^2 = \text{K}$$

$$\therefore \qquad P = N_s^2 \frac{H^{5/2}}{N^2} \qquad \text{or} \qquad N_s^2 = \frac{N^2 P}{H^{5/2}}$$

$$\therefore \qquad N_s = \sqrt{\frac{N^2 P}{H^{5/2}}} = \frac{N\sqrt{P}}{H^{5/4}} \qquad \ldots(18.28)$$

Cavitation

Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapour pressure and the sudden collapsing of these vapour bubbles in a region of higher pressure. When the vapour bubbles collapse, a very high pressure is created. The metallic surfaces, above which these vapour bubbles collapse, is subjected to these high pressures, which cause pitting action on the surface. Thus cavities are formed on the metallic surface and also considerable noise and vibrations are produced.

Cavitation includes formation of vapour bubbles of the flowing liquid and collapsing of the vapour bubbles. Formation of vapour bubbles of the flowing liquid take place only whenever the pressure in any region falls below vapour pressure. When the pressure of the flowing liquid is less than its vapour pressure, the liquid starts boiling and vapour bubbles are formed. These vapour bubbles are carried along with the flowing liquid to higher pressure zones where these vapours condense and bubbles collapse. Due to sudden collapsing of the bubbles on the metallic surface, high pressure is produced and metallic surfaces are subjected to high local stresses. Thus the surfaces are damaged.

Precaution against Cavitation

(*i*) The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure. If the flowing liquid is water, then the absolute pressure head should not be below 2.5 m of water.

(*ii*) The special materials or coatings such as aluminium-bronze and stainless steel, which are cavitation resistant materials, should be used.

Effects of Cavitation

(*i*) The metallic surfaces are damaged and cavities are formed on the surfaces.

(*ii*) Due to sudden collapse of vapour bubble, considerable noise and vibrations are produced.

(*iii*) The efficiency of a turbine decreases due to cavitation. Due to pitting action, the surface of the turbine blades becomes rough and the force exerted by water on the turbine blades decreases. Hence, the work done by water or output horse power becomes less and thus efficiency decreases.

Cavitation in turbines

19.11.4 Cavitation in Turbines. In turbines, only reaction turbines are subjected to cavitation. In reaction turbines the cavitation may occur at the outlet of the runner or at the inlet of the draft-tube where the pressure is considerably reduced (*i.e.*, which may be below the vapour pressure of the liquid flowing through the turbine). Due to cavitation, the metal of the runner vanes and draft-tube is gradually eaten away, which results in lowering the efficiency of the turbine. Hence, the cavitation in a reaction turbine can be noted by a sudden drop in efficiency. In order to determine whether cavitation will occur in any portion of a reaction turbine, the critical value of Thoma's cavitation factor (σ, sigma) is calculated.

...oma's Cavitation factor

Thoma's Cavitation Factor for Reaction Turbines. Prof. D. Thoma suggested a dimensionless number, called after his name Thoma's cavitation factor σ (sigma), which can be used for determining the region where cavitation takes place in reaction turbines. The mathematical expression for the Thoma's cavitation factor is given by

$$\sigma = \frac{H_b - H_s}{H} = \frac{(H_{atm} - H_v) - H_s}{H} \qquad \text{...(19.23)}$$

where H_b = Barometric pressure head in m of water,

H_{atm} = Atmospheric pressure head in m of water,

H_v = Vapour pressure head in m of water,

H_s = Suction pressure at the outlet of reaction turbine in m of water or height of turbine runner above the tail water surface,

H = Net head on the turbine in m.

Cavitation in Centrifugal Pump

19.11.5 Cavitation in Centrifugal Pumps. In centrifugal pumps the cavitation may occur at the inlet of the impeller of the pump, or at the suction side of the pumps, where the pressure is considerably reduced. Hence if the pressure at the suction side of the pump drops below the vapour pressure of the liquid then the cavitation may occur. The cavitation in a pump can be noted by a sudden drop in efficiency and head. In order to determine whether cavitation will occur in any portion of the suction side of the pump, the critical value of Thoma's cavitation factor (σ) is calculated.

Thoma's Cavitation factor

Thoma's Cavitation Factor for Centrifugal Pumps. The mathematical expression for Thoma's cavitation factor for centrifugal pump is given by

$$\sigma = \frac{(H_b) - H_S - h_{LS}}{H} = \frac{(H_{atm} - H_V) - H_S - h_{LS}}{H} \qquad \ldots(19.24)$$

▶ 19.13 NET POSITIVE SUCTION HEAD (NPSH)

The term NPSH (Net Positive Suction Head) is very commonly used in the pump industry. Actually the minimum suction conditions are more frequently specified in terms of NPSH.

The net positive suction head (NPSH) is defined as the *absolute* pressure head at the inlet to the pump, minus the vapour pressure head (in absolute units) plus the velocity head.

∴ NPSH = Absolute pressure head at inlet of the pump – vapour pressure head (absolute units) + velocity head

$$= \frac{p_1}{\rho g} - \frac{p_v}{\rho g} + \frac{v_s^2}{2g} \quad (\because \text{ Absolute pressure at inlet of pump} = p_1) \quad \text{... (19.32)}$$

But from equation (*ii*) of Art. 19.12, the absolute pressure head at inlet of the pump is given by as

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)$$

Substituting this value in equation (19.32) , we get

$$\text{NPSH} = \left[\frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)\right] - \frac{p_v}{\rho g} + \frac{v_s^2}{2g}$$

$$= \frac{p_a}{\rho g} - \frac{p_v}{\rho g} - h_s - h_{f_s}$$

$$= H_a - H_v - h_s - h_{f_s}$$

$$\left(\because \frac{p_a}{\rho g} = H_a = \text{Atmospheric pressure head}, \frac{p_v}{\rho g} = H_v = \text{Vapour pressure head}\right)$$

$$= \left[\left(H_a - h_s - h_{f_s}\right) - H_v\right] \quad \text{...(19.33)}$$

The right hand side of equation (19.33) is the total suction head. Hence NPSH is equal to total suction head. Thus NPSH may also be defined as the total head required to make the liquid flow through the suction pipe to the pump impeller.

References

- **A Textbook of Fluid Mechanics and Hydraulic Machines**
Dr. R. K. Bansal, Laxmi Publications

- **NPTEL VIDEO LECTURES**